Last lecture: Ion traps

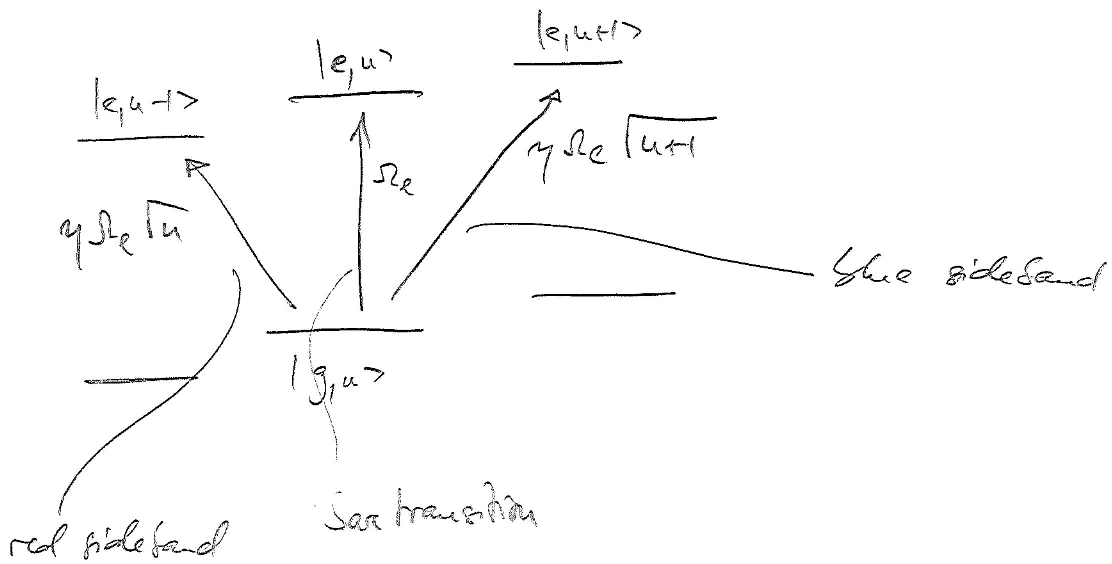
Trap ions in linear Paul trap

Weak confinement in z-direction

$\rightarrow$ vibrational mode in z-direction: quantize H.O.: $a, a^\dagger$

Two-level in trap + laser: coupling of int. levels & vibrational mode:

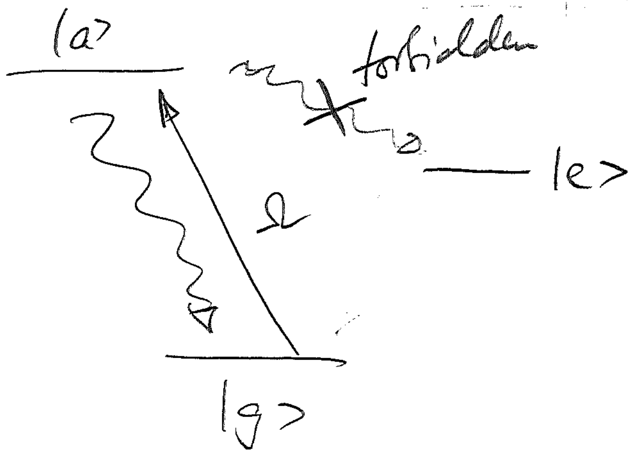
$$H_{cpl} = i\hbar \frac{\Omega_e}{2} e^{-i\omega_c t}\left(1 - i\eta(a^\dagger + a)\right)\sigma_+ + h.c.$$



Initialization: optical pumping via 3rd level

Cooling: Doppler cooling + sideband cooling

Read-out of encoded information (i.e., $|g\rangle$ or $|e\rangle$):



Use fast-decaying 3rd level coupled to $|g\rangle$ via strong laser

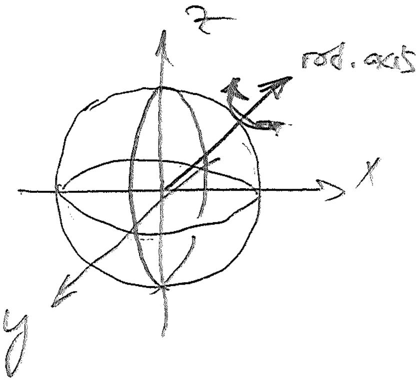
$\Rightarrow$ fluorescence if & only if atom in $|g\rangle$.

Short lifetime + strong laser $\Rightarrow$ strong signal (up to 10^8 photons/sec!)

Tomography:

Can we fully determine the state ρ of the two-level system (=qubit)?

* Repeated meas. determines $\mathrm{tr}[\rho\sigma_z]$.



* Ramsey pulses (→ pg. 45/46): rotate qubit s.th. x or y axis are mapped to z
  → measurement of $\mathrm{tr}[\rho\sigma_x]$, $\mathrm{tr}[\rho\sigma_y]$

$\Rightarrow$ allows to reconstruct $\rho = \dfrac{\mathbb{1} + \vec{v}\cdot\vec{\sigma}}{2}$

using that $v_x = \mathrm{tr}[\rho\sigma_x]$, $v_y = \mathrm{tr}[\rho\sigma_y]$, $v_z = \mathrm{tr}[\rho\sigma_z]$.

Engineering of oscillator states:

Atom-phonon interaction can be used to engineer phonon states.

Example 1: Fock states $|n\rangle$

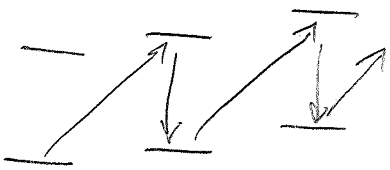
Start in $|g,0\rangle$ and use sequence of

(i) blue sideband trans. (π-pulse)

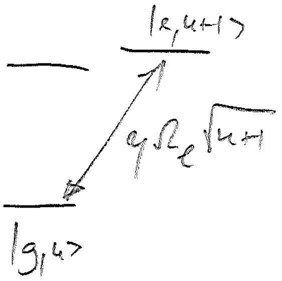
(ii) bare (or red SB) π-pulses

$$|g,0\rangle \xrightarrow{(i)} |e,1\rangle \xrightarrow{(ii)} |g,1\rangle \longrightarrow |e,2\rangle \longrightarrow |g,2\rangle \rightarrow \ldots$$

$$\left(\text{or } |g,0\rangle \xrightarrow{(i)} |e,1\rangle \xrightarrow{(ii)} |g,2\rangle \rightarrow \ldots \right)$$



Read-out: Induce Rabi osc. on blue sideband:



* Rabi freq. $\propto \sqrt{n+1}$
* meas. occupation of $|g\rangle$ as func. of time

$\Rightarrow$ phonon counting meas.!

Example 2: Prep. of arbitrary states $|\psi\rangle = \sum_{n=0}^{N} c_n |n\rangle$.

- Aim: Prepare $|\psi\rangle$ from $|g,0\rangle$ by pulse seq.
- Easier: find pulse seq. to transform $|\psi\rangle$ into $|g,0\rangle$ and invert it.

Iterative procedure:

- Initial state is of the form

$$\sum_{n=0}^{N} \lambda_{g,n} |g,n\rangle + \sum_{n=0}^{N-1} \lambda_{e,n} |e,n\rangle \qquad (*)$$

(i) Apply red SB pulse which maps

$$\lambda_{g,N} |g,N\rangle + \lambda_{e,N-1} |e,N-1\rangle \longrightarrow \text{const} \times |e,N-1\rangle$$

State is now of the form

$$\sum_{n=0}^{N-1} \tilde{\lambda}_{g,n} |g,n\rangle + \sum_{n=0}^{N-1} \tilde{\lambda}_{e,n} |e,n\rangle$$

with known $\tilde{\lambda}_{s,n}$.

(ii) Apply pulse on free transition s.th.

$$\tilde{\lambda}_{g,N-1} |g,N-1\rangle + \tilde{\lambda}_{e,N-1} |e,N-1\rangle \rightarrow \text{const} \times |g,N-1\rangle$$

$\Rightarrow$ State is now again of the form $(*)$.

$\longrightarrow$ iterate (i) + (ii) until state is $|g,0\rangle$ (i.e. $N=0$).

Several ions in a trap:

Consider N ions in a linear trap:

Ions arrange in chain. Ham. around equilibrium is

$$H = \frac{1}{2M} \sum_{u=1}^{N} p_u^2 + \frac{1}{2} M\nu^2 \sum A_{mu} q_u q_m \,, \quad A_{mu} = A_{um} .$$

Diagonalize A_{mu}: $\quad A_{mu} = \sum_k \mu_k b_m^k b_u^k \quad$ (b_m^k form ONB):

$\Rightarrow$ normal mode Ham.

$$H = \frac{1}{2M} \sum_{k=1}^{N} \tilde{P}_k^2 + \frac{1}{2} M\nu^2 \sum_{k=1}^{N} \mu_k \tilde{q}_k^2 \,;$$

$\tilde{p}_k = \sum_u b_u^k p_u \,; \; \tilde{q}_k = \sum_u b_u^k q_u$ canonical variables $\Rightarrow$ quantize!

$\Rightarrow$ N indep. H.O.s with position $\tilde{x}_k = \sum_u b_u^k x_u$ and freq. $\sqrt{\mu_k}\,\nu$. Each mode couples to pos. of all ions!

E.g.: 2 ions: $\mu_1 = 1$; $\tilde{x}_1 = \frac{1}{\sqrt{2}}(x_1 + x_2)$: "center-of-mass (CM) mode"

$\mu_2 = \sqrt{3}$; $\tilde{x}_2 = \frac{1}{\sqrt{2}}(x_1 - x_2)$: "breathing mode"

$\tilde{x}_k = \sum_u b_u^k x_u \Leftrightarrow x_u = \sum_k b_u^k \tilde{x}_k$ (ONB) $\Rightarrow$ apply laser:

$$H_{cpl} = \frac{i\hbar\Omega_e}{2} e^{-i\omega_L t} (1 + i\eta \tilde{x}_u) + h.c. =$$

$$= \frac{i\hbar\Omega_e}{2} e^{-i\omega_L t} \left(1 + i\eta \left(\sum_k b_u^k a_0 (a_k^\dagger + a_k)\right)\right) + h.c.$$

$\Rightarrow$ each ion couples to <u>all</u> normal modes

Normal modes have different freqs. $\Rightarrow$ indiv. mode can be <u>selected</u> by tuning ω_e!

<u>Note</u>: (i) Spacing of ions ~ $1\mu m$ $\Rightarrow$ can be addressed indiv. by lasers.
(ii) Spacing of normal mode freqs. dep. on # ions $\Rightarrow$ limitations!

<u>Ion trap quantum computing</u>

<u>Quantum computer</u>:

	in ion trap
◦ N qubits (= two-level systems); states $\|0\rangle \equiv \|g\rangle$, $\|1\rangle \equiv \|e\rangle$.	✓
◦ ability to initialize qubits (e.g. to $\|0\rangle$)	✓
◦ ability to perform arb. single qubit operations (= Bloch sphere rotations)	✓ (Ramsey pulses)
◦ ability to read out qubit (e.g. in Z basis)	✓
◦ entangling 2-qubit gate, e.g. controlled-Z (C-Z):	

$$
\begin{aligned}
CZ:\ &|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle \\
&|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle \\
&|1\rangle|0\rangle \rightarrow |1\rangle|0\rangle \\
&|1\rangle|1\rangle \rightarrow -|1\rangle|1\rangle
\end{aligned}
$$

$\rightarrow$ Use vibrational mode as "quantum data bus"

(◦ Note: Also need <u>scalability</u>; more in Ihle n ion traps)

Cirac-Zoller gate:

CZ betw. pair of qubits (denoted by 1 & 2 wlog)

Idea: (i) Transfer state of ① onto CM mode

(ii) perform CZ between ② and CM mode

(iii) transfer state from CM mode back onto ①.

Sufficient to analyze protocol on basis states (linearity!)

$$|g,g\rangle, \; |g,e\rangle, \; |e,g\rangle, \; |e,e\rangle$$

(as long as we use the same global phase for all of them!)

We start with vibrational ground state $|0\rangle$.

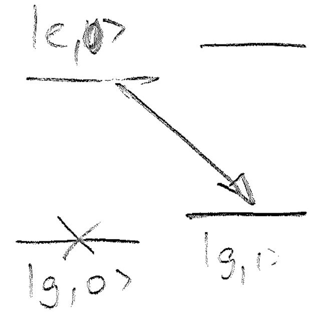
(i) transfer ① onto CM mode:

red SB π-pulse on ①:

$$\overset{①}{} \quad \overset{②}{} \quad \overset{\text{vibr.}}{}$$

$$|g_1, \alpha_2; 0\rangle \longrightarrow |g_1, \alpha_2; 0\rangle$$

$$|e_1, \alpha_2; 0\rangle \longrightarrow |g_1, \alpha_2; 1\rangle$$



(ii) CZ betw. ② and CM mode:

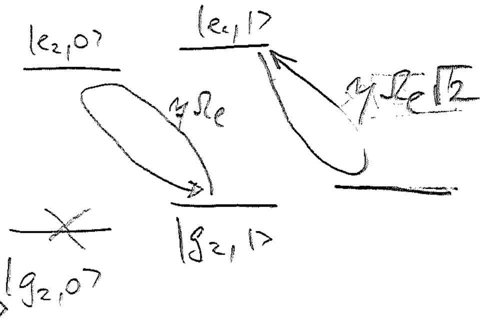
1st idea: Use 2π pulse on red SB (note: 2π pulse gives (−1) phase, cf. lecture 6)

$$|\alpha_1, g_2; 0\rangle \longrightarrow |\alpha_1, g_2; 0\rangle$$

$$|\alpha_1, g_2; 1\rangle \longrightarrow -|\alpha_1, g_2; 1\rangle$$

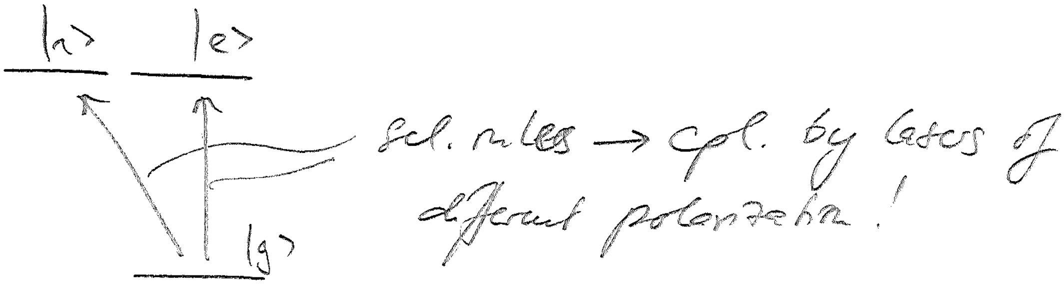
$$|\alpha_1, e_2; 0\rangle \longrightarrow -|\alpha_1, e_2; 0\rangle$$

$$|\alpha_1, e_2; 1\rangle \longrightarrow \cos(\sqrt{2}\pi)\,|\alpha_1, e_2; 1\rangle + \sin(\sqrt{2}\pi)\,|\alpha_1, g_2; 2\rangle$$



Problem: System "leaks" into $|g_2, 2\rangle$, since Rabi freq. dep. on n!

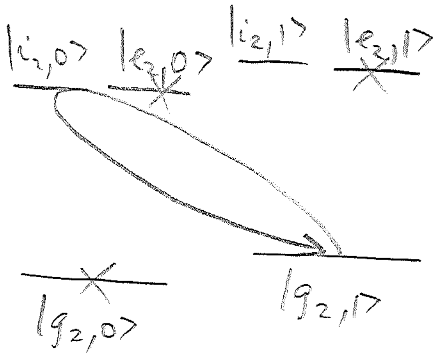
Solution: Use 3rd level:



2π-pulse on red SB of $|g\rangle \leftrightarrow |i\rangle$ transition:

→ similar as before, but $|e_2;0\rangle$ & $|e_2;1\rangle$ do not evolve!

$$|\alpha_1, g_2; 0\rangle \longrightarrow |\alpha_1, g_2; 0\rangle$$
$$|\alpha_1, g_2; 1\rangle \longrightarrow -|\alpha_1, g_2; 1\rangle$$
$$|\alpha_1, e_2; 0\rangle \longrightarrow |\alpha_1, e_2; 0\rangle$$
$$|\alpha_1, e_2; 1\rangle \longrightarrow |\alpha_1, e_2; 1\rangle$$



(iii) transfer CM mode into (1):

red SB π pulse on (1):

$$|g_1, \alpha_2; 0\rangle \longrightarrow |g_1, \alpha_2; 0\rangle$$
$$|g_1, \alpha_2; 1\rangle \longrightarrow -|e_1, \alpha_2; 0\rangle$$

Total sequence:

$$
\begin{aligned}
|g_1, g_2; 0\rangle &\xrightarrow{(i)} |g_1, g_2; 0\rangle \xrightarrow{(ii)} |g_1, g_2; 0\rangle \xrightarrow{(iii)} |g_1, g_2; 0\rangle \\
|g_1, e_2; 0\rangle &\longrightarrow |g_1, e_2; 0\rangle \longrightarrow |g_1, e_2; 0\rangle \longrightarrow |g_1, e_2; 0\rangle \\
|e_1, g_2; 0\rangle &\longrightarrow |g_1, g_2; 1\rangle \longrightarrow -|g_1, g_2; 1\rangle \longrightarrow |e_1, g_2; 0\rangle \\
|e_1, e_2; 0\rangle &\longrightarrow |g_1, e_2; 1\rangle \longrightarrow -|g_1, e_2; 1\rangle \longrightarrow -|e_1, e_2; 0\rangle
\end{aligned}
$$

⇒ Controlled-Z gate!

⇒ Quantum computing scheme with ion traps!