VI. Ion traps

* coherent manipulation of individual quantum systems

  $\longrightarrow$ well isolated q. system over long time

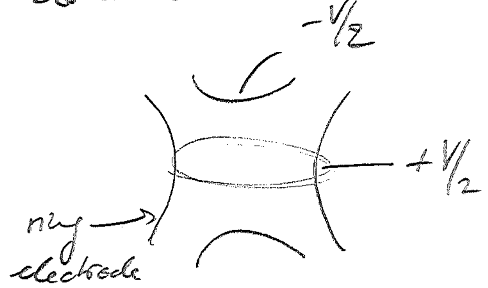
* Ion traps: trap charged particles using electric/magnetic fields – ess. arbitrarily long storage times (several months)

How to trap charged particles?

- Electric potential $U(\vec{r})$ w/ minimum?
- Impossible: $\Delta U(\vec{r}) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) U(\vec{r}) = 0$ (w/out charges)

  $\Longrightarrow$ only saddle point can be obtained:

  $$U(\vec{r}) = \alpha\left(x^2 + y^2 - 2z^2\right)$$



- How can we stabilize 3rd direction?

* Penning trap:
  - Use field to confine in the z direction
  - in XY direction: magnetic field along z

    $\Longrightarrow$ stable cyclotron orbit

* Paul trap:

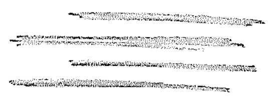
- no magnetic field
- time-dep. V: $V(t) = V_0 \cos(\omega_{rf} t)$

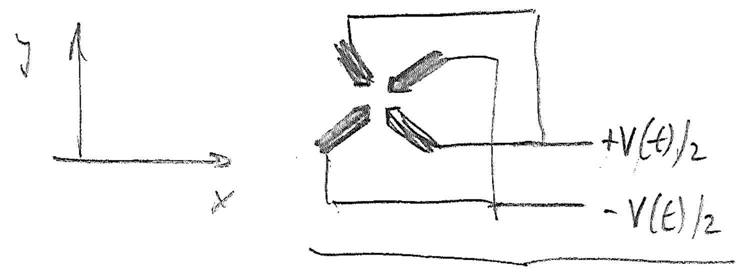
  typ: $\frac{\omega_{rf}}{2\pi} \sim 10\text{-}100\,\text{MHz}$; $V \sim 100\,\text{V}$; trap size $\sim 1\,\text{mm}$

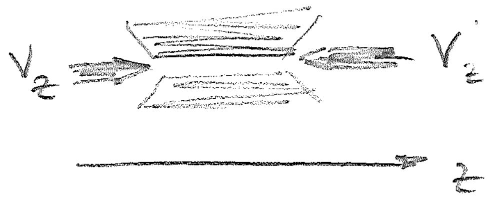
- potential oscillates betw. trapping & anti-trapping in all directions
- stable orbit (as long as ion far enough from bnd.)

Variant of Paul trap:

Linear trap ($\Rightarrow$ will consider this in the following)

4 electrodes:  + 2 electrodes in z direction:





$V(t) = V_{xy} \cos(\omega_{rf} t)$

- Strong confining potential along xy
- weak conf. potential (harmonic trap) along z

  $\Rightarrow$ oscillations in z direction can be excited

- Linear shape: several ions can be placed in trap (along z axis)

Which ions to use?

- simple level structure: one valence electron in ion
  $\rightarrow$ e.g. 2nd group, singly charged (e.g. Ca^+, Be^+)

Quantization of vibration along z:

Ham. of ion (no internal levels) in trap:

$$H_{trap} = \frac{\hat{p}^2}{2M} + \frac{1}{2} M \nu^2 \hat{z}^2$$

M: mass of ion

ν: freq. of z oscillation (typ. $\frac{\nu}{2\pi} \sim 0.5\,\mathrm{MHz} \ldots 10\,\mathrm{MHz}$)

Quantization:

$$\hat{z} = \sqrt{\frac{\hbar}{2M\nu}}\,(a + a^+)$$

$$\hat{p} = i\sqrt{\frac{\hbar M \nu}{2}}\,(a^+ - a)$$

$$\Rightarrow H_{trap} = \hbar\nu\left(a^+ a + \frac{1}{2}\right)$$. excitations: "phonons"

Trap oscillation quantized.

Note: Energy very small compared to optical trans. ($\sim 10^{15}\,\mathrm{Hz}$)

Two-level atom in trap:

Consider 2-level atom (levels $|g\rangle, |e\rangle$, trans. ω_{eg}) in lin. trap:

$$H_0 = \underbrace{\frac{\hbar\omega_{eg}}{2}\sigma_z}_{H_{atom}} + \underbrace{\hbar\nu\left(a^\dagger a + \frac{1}{2}\right)}_{H_{trap}}$$

Shine strong classical beam at freq. ω_e on ion:

Coupling via electric dipole Ham.

$$H_{cpl} = -\vec{d}\cdot\vec{E}(r) \quad ; \quad \vec{d}: \text{dipole moment}$$

$$\vec{E}(r) = \vec{\mathcal{E}}\, e^{i(k_e r - \omega_e t)} \quad \text{depends on } \underline{\text{position}}$$

$\underline{\text{of ion}} \Rightarrow$ coupling to oscillation of ion

(replace z-component of $\vec{r}$: $z \to a_0(a+a^\dagger)$,

$a_0 = \sqrt{\frac{\hbar}{2M\nu}}$ size of zero-point motion)

$$\Rightarrow H_{cpl} = -i\hbar\frac{\Omega_e}{2}e^{-i\omega_e t}e^{ik_e z\cos\Theta}\sigma_+ + h.c.$$

Ω_e: class. Rabi freq. given by d & $\mathcal{E}$ (cf. pg. 44)

Θ: angle betw. laser & z-axis

With $z \to a_0(a+a^\dagger)$:

$$e^{ik_e z\cos\Theta} = e^{i\eta(a+a^\dagger)},$$

with $\eta = \eta_0\cos\Theta$,

and $\underline{\eta_0 = k_e a_0}$ the $\boxed{\text{Lamb-Dicke parameter}}$

η_0 measures extension of G.S. of oscillator relative to wavelength $2\pi/k_e$ of light.

We are interested in regime where

$\eta_0 \ll 1$ ("Lamb-Dicke regime")

Typ. experiments: $\eta_0 = 0.05 \ldots 0.3$.

<u>Note</u>: By tuning Θ, any $0 \leq \eta \leq \eta_0$ can be achieved.

Expand H_{cpl} in orders of η:

$$e^{i\eta(a+a^\dagger)} = 1 + i\eta(a+a^\dagger) + O(\eta^2 \underbrace{(a+a^\dagger)^2}_{\sim n^2})$$

→ expansion in fact in $\eta\langle a^\dagger a\rangle \ll 1$!

$$\Rightarrow H_{cpl} \approx -i\hbar \frac{\Omega_e}{2} e^{-i\omega_e t}\left(1 + i\eta(a+a^\dagger)\right)\sigma_+ + h.c.$$

In interaction picture w.r.t. H_0, we find:

$$H_{cpl} = H_{bare} + H_{red} + H_{blue}, \quad \text{with}$$

* $H_{bare} = -i\hbar \frac{\Omega_e}{2}\left(e^{i(\omega_{eg}-\omega_e)t}\sigma_+ + h.c.\right)$

normal Rabi oscillation in class. field

("bare transition")

* $H_{red} = \hbar \eta \frac{\Omega_e}{2} \left(e^{i(\omega_{eg}-\omega_e-\nu)} a \sigma_+ + h.c. \right)$

"red sideband transition" —

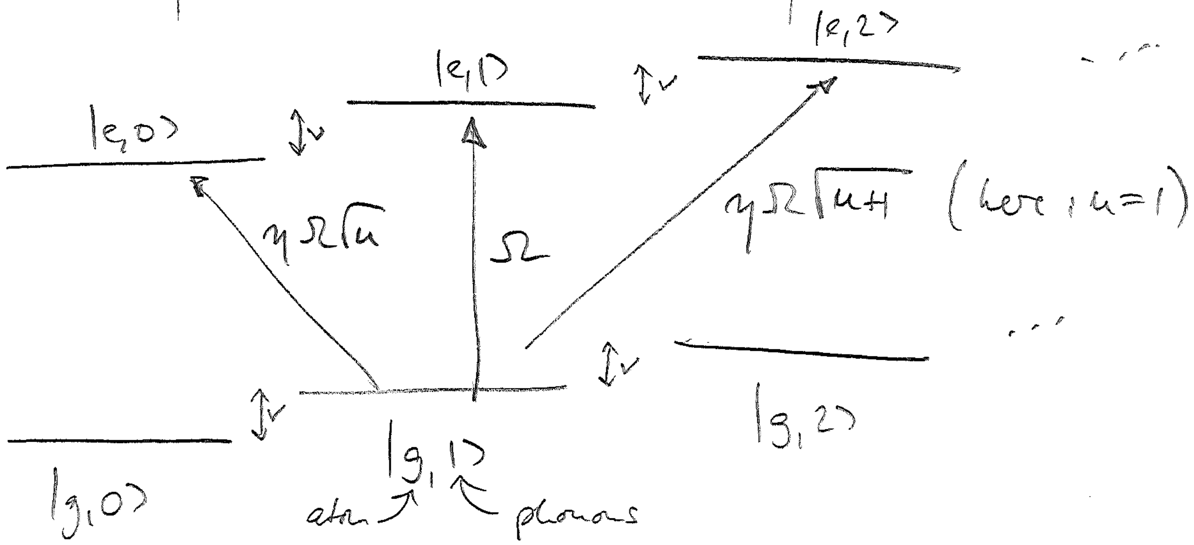
Jaynes-Cummings-type cpl. between atom & oscillator mode

* $H_{blue} = \hbar \eta \frac{\Omega_e}{2} \left(e^{i(\omega_{eg}-\omega_e+\nu)} a^\dagger \sigma_+ + h.c. \right)$

"blue sideband transition" —

anti-Jaynes-Cummings model.

	frequency of trans.	transition rate ≙ Rabi freq.
bare trans.	$\omega_e = \omega_{eg}$	Ω
red sideband	$\omega_e = \omega_{eg} - \nu$	$\eta \Omega a \to \eta \Omega \sqrt{n}$
blue sideband	$\omega_e = \omega_{eg} + \nu$	$\eta \Omega a^\dagger \to \eta \Omega \sqrt{n+1}$



Note: • Excit. of sideband requires that we are strongly detuned from bare transition: $\Omega \ll \nu$ (strong constr., as ν very small)

• It also requires narrow bandwidth (≙ long lifetime) of $|e\rangle \leftrightarrow |g\rangle$ transition!

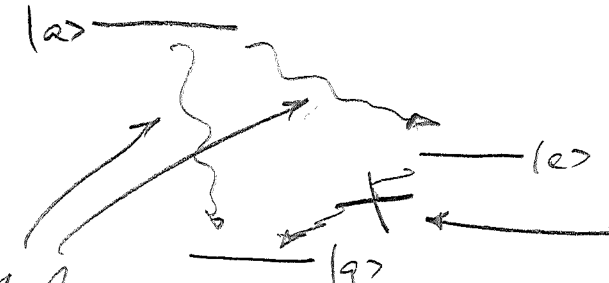
Note: If $|e\rangle \leftrightarrow |g\rangle$ is a dipole-forbidden transition ($\Leftrightarrow$ long lifetime, i.e., desirable!), the same H_{cpl} can also be implemented using (i) quadrupole transitions or (ii) a Raman scheme cpl. $|g\rangle$ and $|e\rangle$ via a 3rd level.
(Ω_e needs to be replaced by the proper Rabi freq. in both cases.)

Note: If we use a standing wave, we can tune the rel. strength of carr. vs. sideband trans. using the phase:

$$E(\vec{r}) = \vec{\mathcal{E}}\left(\sin(kz+\phi)\, e^{-i\omega_L t}\right)$$

$$\Rightarrow H_{int} = -i\hbar \frac{\Omega_e}{2}\left(\sin\phi + \eta\cos\phi\,(a+a^\dagger)\right) e^{-i\omega_L t} + h.c.$$

Beyond the levels $|g\rangle$ and $|e\rangle$, more level(s) are needed for initialization, cooling, and read-out:



dipole-forbidden: long lifetime
$\Longrightarrow$ information storage

short lifetime:
preparation, read-out

Preparation: Optical pumping, e.g. via $|e\rangle \leftrightarrow |a\rangle$ laser
$\Rightarrow$ transition to $|a\rangle$ w/ decay to either $|e\rangle$ or $|g\rangle$
$\Longrightarrow$ preparation into $|g\rangle$!

Cooling: Energy of one oscillator quantum: $\hbar\nu$, $\frac{\nu}{2\pi} \sim 1\,\mathrm{MHz}$;

Initial energy of ions ~ trap depth ~ 100 eV

$\Rightarrow$ over 10^9 phonons!

$\Rightarrow$ cooling of vibrational mode required to get into ground state!

Two-step procedure:

1) Doppler cooling:

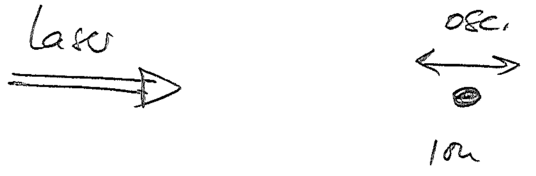
Use a very short-lived transition $|g\rangle \leftrightarrow |a\rangle$ (typ. $\Gamma \sim 10^8\,s^{-1}$)

$\Rightarrow$ Large linewidth $\Gamma \gg \nu$

$\Rightarrow$ sidebands (phonons) cannot be resolved

$\Rightarrow$ oscillator degree of freedom ess. classical

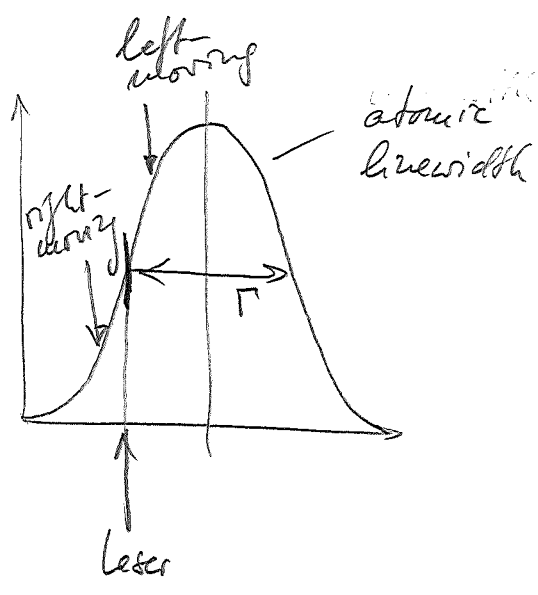
Use laser detuned by $\Gamma/2$ from ω_{ag}:



Atom absorbs & emits photons rapidly.

Emission: isotropic = no net momentum transfer.

Absorption: larger for left-moving ion $\Rightarrow$ net decrease of momentum $\Rightarrow$ cooling



$\boxed{\text{Doppler cooling}}$

Limit to Doppler cooling:

Fluctuations from:

* fluct. photon # in beam (= absorbed photons)
* emission in rand. direction

⇒ limitation in Doppler cooling

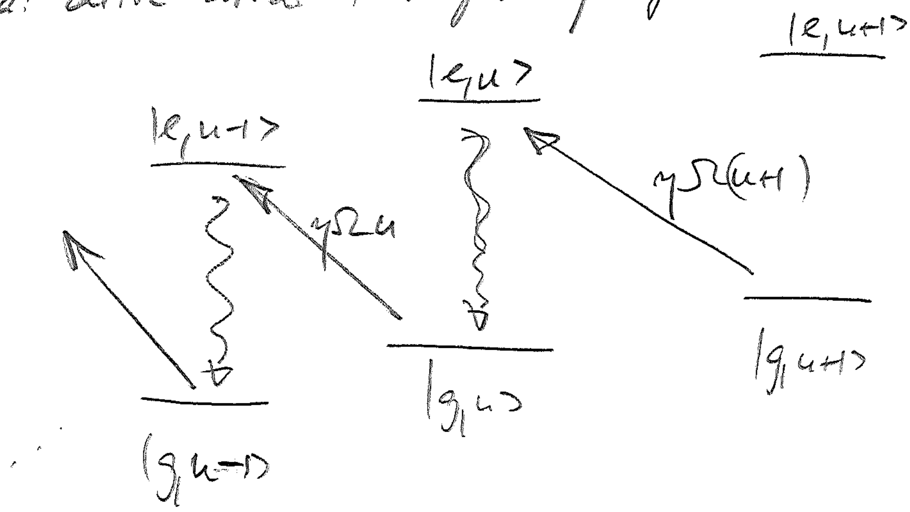
$$n_{th} \sim \frac{\Gamma}{\nu} \text{ phonons left}$$

(⇒ book by Haroche + Raimond)

⇒ Second cooling step necessary.

2. Sideband cooling:

Idea: drive atom through seq. of red sideband transitions:



Step 1: Laser at red sideband transition:

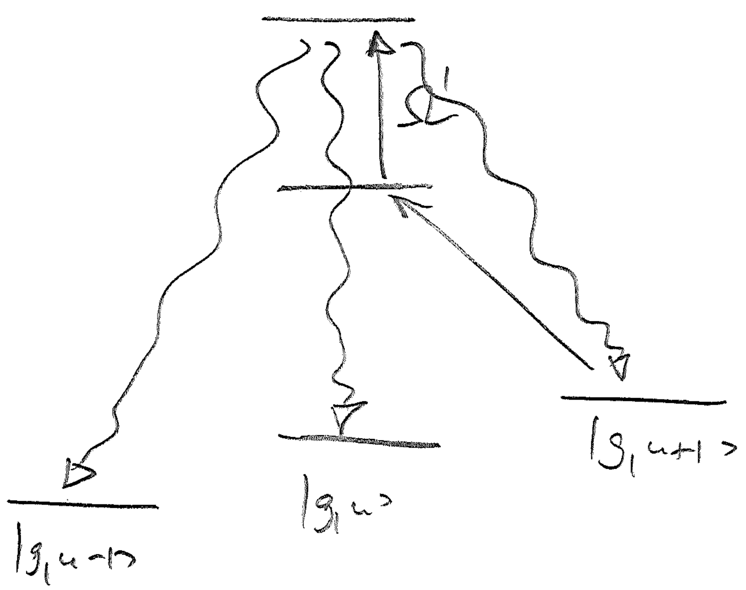
$$|g, n+1\rangle \longleftrightarrow |e, n\rangle$$

Requirement: Sidebands can be resolved ⇒ linewidth of $|e\rangle \leftrightarrow |g\rangle$ trans. (& of laser) $\underline{\Gamma_{eg} \ll \nu}$! (strong req.!)

<u>Step 2</u>: Need rapid decay $|e,n\rangle \to |g,n\rangle$.

<u>But</u>: Γ_{eg} <u>small!</u>

$\Rightarrow$ Use 2nd laser to excite $|e\rangle$ to short-lived level (cf. HW Sheet 11, Problem 2):



Decay via sidebands suppressed by $\eta \cdot n$ $\Rightarrow$ decay into $|g,n\rangle$ most likely $\Rightarrow$ cooling down to vibr. G.S.

$|g,0\rangle$ "dark state" $\longrightarrow$ stable

<u>Measurement of cooling:</u> Compare fluorescence of red vs. blue sideband $\Rightarrow$ occupation of $|g,0\rangle$.

Ground state can be prepared w/ > 99% prob.