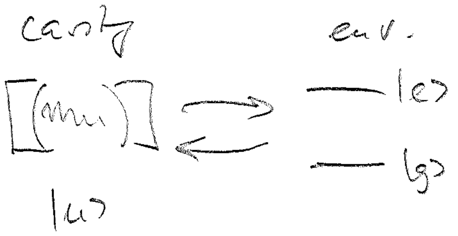
Addendum (to pg. 79):

Argument for transition rate in cpl. to thermal bath:

1) Toy model: Env. has only 2 levels



absorption: rate $\kappa_+(n+1)$

$$|n, e\rangle \rightleftarrows |n+1, g\rangle$$

emission: rate $\kappa_-(n+1)$

Ratio of rates = ratio of prob. to find env. in $|g\rangle$ or $|e\rangle$:

$$\frac{\kappa_+}{\kappa_-} = \frac{\text{prob(env in } e)}{\text{prob(env in } g)} = e^{-\hbar\omega/k_B T}.$$

↑ thermal eq.

2) Realistic environment: Env. has many levels $|i\rangle$:

Emission / absorption involves pair of levels $|\alpha_g\rangle, |\alpha_e\rangle$ with $E - E = \hbar\omega$ — but many such levels can exist

$$\frac{\kappa_+}{\kappa_-} = \frac{\text{prob(env. in any of the } \alpha_e\text{'s)}}{\text{prob(env. in any of the } \alpha_g\text{'s)}} = \frac{\sum p(\alpha_e)}{\sum p(\alpha_g)}$$

$$= \frac{\sum e^{-\hbar\omega/k_B T} p(\alpha_g)}{\sum p(\alpha_g)} = e^{-\hbar\omega/k_B T}.$$

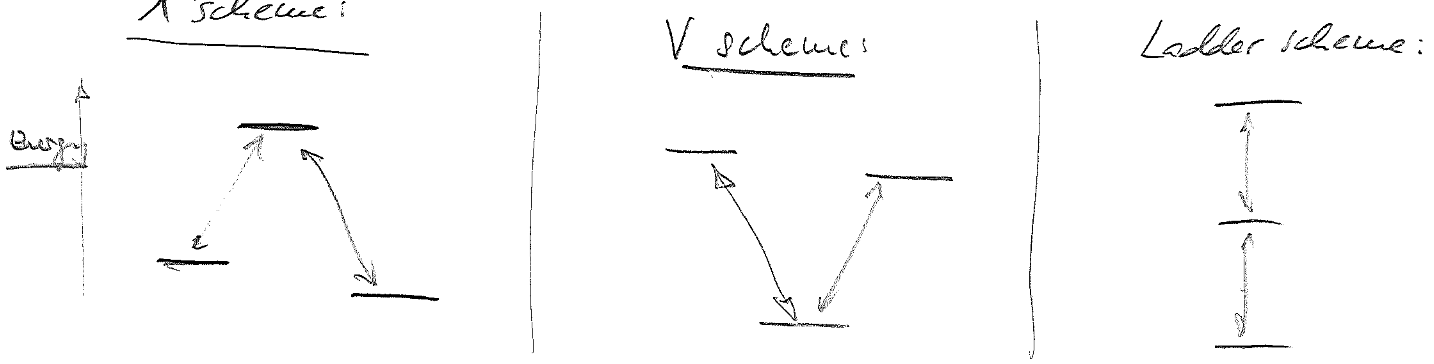
V. Interaction of multi-level systems w/ light

Two-level system was strong simplification – atoms have many levels.

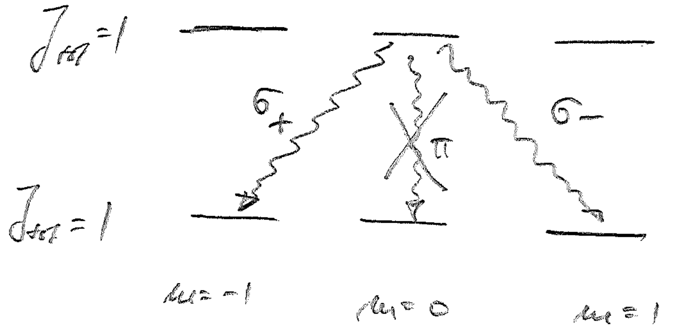
Can we get interesting physics by using more levels?

Focus on three-level systems:

Many different configurations:



E.g.:

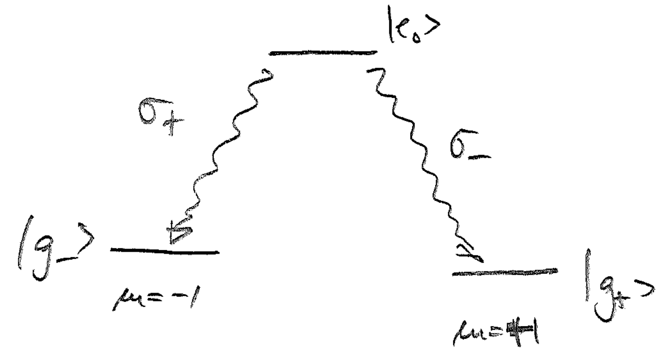


σ_+ / σ_-: transition w. circ. polarized photon

π: linearly polarized photon.

$(m = 0) \to (m = 0)$ transition can be forbidden by selection rules in many-electron system $\Rightarrow$ effective 3-level system

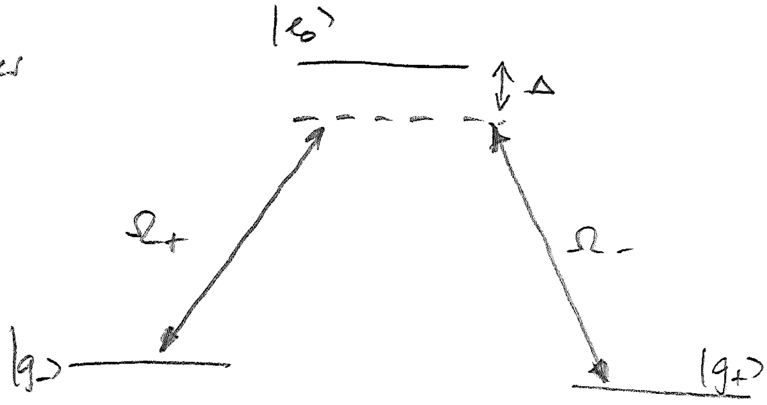
Λ scheme:



transition $|g_-\rangle \longleftrightarrow |g_+\rangle$ forbidden.

Can we reduce trans. via $|e_0\rangle$?

Apply σ_+/σ_- polarized laser beams ($\equiv$ classical light) w/ detuning Δ & strengths characterized by coupling constants $\Omega_\pm$ in Jaynes-Cummings model.



(i.e.: Freq. $\omega_\pm$ s.th. $\hbar\omega_\pm + \Delta = E_e - E_{g_\mp}$)

Hamiltonian in interaction picture:

$$H = -\hbar\Delta\,|e_0\rangle\langle e_0| - i\hbar\frac{\Omega_-}{2}\left(|e_0\rangle\langle g_+| - |g_+\rangle\langle e_0|\right) - i\hbar\frac{\Omega_+}{2}\left(|e_0\rangle\langle g_-| - |g_-\rangle\langle e_0|\right)$$

Schrödinger equation: $|\psi\rangle = c_0|e_0\rangle + c_+|g_+\rangle + c_-|g_-\rangle$:

$$i\hbar\begin{pmatrix}\dot c_0\\ \dot c_+\\ \dot c_-\end{pmatrix} = \underbrace{\hbar\begin{pmatrix}-\Delta & -i\Omega_-/2 & -i\Omega_+/2\\ i\Omega_-/2 & 0 & 0\\ i\Omega_+/2 & 0 & 0\end{pmatrix}}_{\equiv H}\begin{pmatrix}c_0\\ c_+\\ c_-\end{pmatrix}$$

Strong detuning regime:

c_0 is only weakly populated $\Rightarrow \dot{c}_0 \approx 0$

$$\Rightarrow \quad 0 = -\Delta c_0 - \frac{i\Omega_-}{2} c_+ - \frac{i\Omega_+}{2} c_-$$

$$\Rightarrow \quad c_0 = -i \, \frac{\Omega_- c_+ + \Omega_+ c_-}{2\Delta}$$

$$\Rightarrow \quad i\hbar \dot{c}_+ = \hbar \frac{i\Omega_-}{2} c_0 = \hbar \left(\frac{\Omega_-^2}{2\Delta} c_+ + \frac{\Omega_+ \Omega_-}{2\Delta} c_- \right)$$

$$i\hbar \dot{c}_- = \hbar \left(\frac{\Omega_+ \Omega_-}{2\Delta} c_+ + \frac{\Omega_+^2}{2\Delta} c_- \right)$$

$\Rightarrow$ Rabi oscillations betw. $|g_+\rangle$ and $|g_-\rangle$ w/ freq. $\frac{\Omega_+ \Omega_-}{2\Delta}$!

* Effective interaction betw. $|g_+\rangle$ and $|g_-\rangle$ mediated by off-resonant interaction w/ excited state $|e\rangle$.

* $|e\rangle$ is never much populated $\Rightarrow$ low decay rate (once $|e\rangle$ decays quickly, $|g_\pm\rangle$ are (meta-) stable).

Raman scheme / transition / process

Beyond strong detuning:

Diagonalize full H:

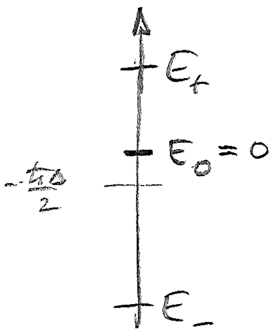
Eigenvalues: $E_0 = 0$

$$E_\pm = -\frac{\hbar\Delta}{2} \pm \sqrt{\Delta^2 + \Omega_{eff}^2} \; ; \quad \Omega_{eff} = \sqrt{\Omega_+^2 + \Omega_-^2}$$

Eigenstates:

$$|0\rangle = \frac{1}{\Omega_{eff}} \left[\Omega_+ |g_+\rangle - \Omega_- |g_-\rangle \right]$$

$$|\pm\rangle = \frac{1}{\sqrt{E_\pm^2 + \Omega_{eff}^2}} \left[\Omega_+ |g_-\rangle + \Omega_- |g_+\rangle - i E_\pm |e_0\rangle \right]$$



$|0\rangle$: <u>"dark state"</u>: it has no overlap w/ $|e\rangle$ ⇒ it has no overlap w/ optical transitions in H ($\equiv |e_0\rangle\langle g_\pm| + h.c.$), and does not decay under emission of (visible) light.

⇒ Atom prepared in $|0\rangle$ will stay in that state (despite the presence of fields $\Omega_\pm$).

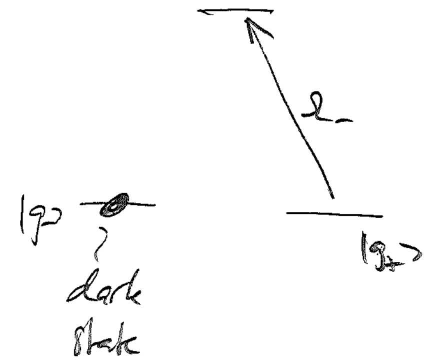
$|0\rangle$ <u>independent of detuning</u> Δ

$|0\rangle$ depends on ratio Ω_+/Ω_-:

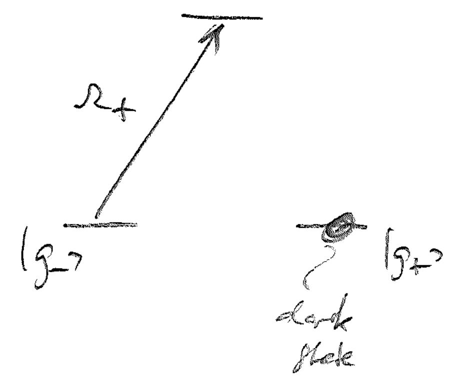
$$|0\rangle \rightarrow |g_+\rangle \text{ for } \Omega_+/\Omega_- \rightarrow \infty$$

$$|0\rangle \rightarrow -|g_-\rangle \text{ for } \Omega_+/\Omega_- \rightarrow 0$$

Ω_+ off:



Ω_- off:



Adiabatic evolution: If system is in eigenstate of $H(t)$ and $H(t)$ changes very slowly (adiabatically), the system follows this eigenstate adiabatically.

Use to transfer system from $|g_-\rangle$ to $|g_+\rangle$ adiabatically.

$t=0$: • system initially in $|g_-\rangle$, laser off

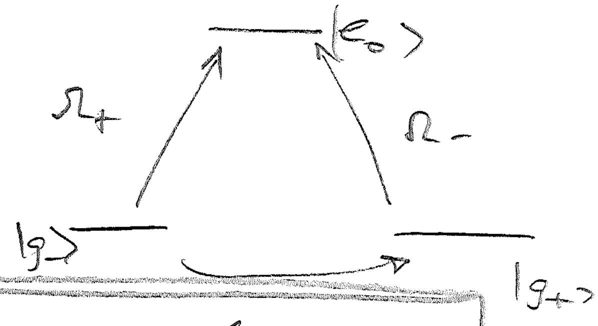
• switch on Ω_- adiabatically
  $\rightarrow$ system stays in $|g_-\rangle \equiv |0\rangle$ (eigenstate)

• adiabatically switch on Ω_+, while decreasing Ω_-
  $\rightarrow$ system follows eigenstate $|0\rangle \propto \Omega_+|g_+\rangle - \Omega_-|g_-\rangle$
  (only ratio of Ω_+/Ω_- matters!)

• finally: $\Omega_- = 0$, Ω_+ on $\Rightarrow$ system in $|g_+\rangle$

$t=\tau$ • switch off Ω_+: system stays in eigenstate $|g_+\rangle$

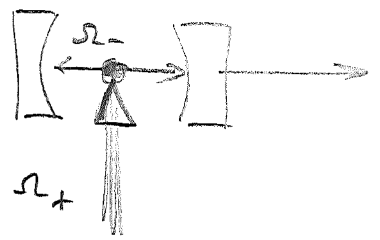
transfer system from $|g_-\rangle$ to $|g_+\rangle$



Stimulated Raman adiabatic passage (STIRAP)

* Can be used to prepare atoms in a specific state

* Useful to "deposit" photons into cavities (cf. Purcell effect):



- Ω_+: strong ext. laser (class. light)

- atom - cavity cpl. via Ω_-

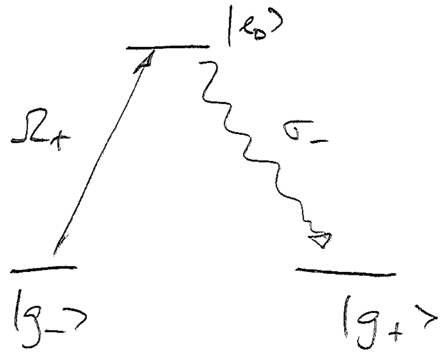
(Note: i) STIRAP also works for quant. light, Rabi freq. $\equiv \Omega_n = \Omega_0 \sqrt{n+1}$

ii) atom also cpl. to vacuum in all directions, but cpl. to cavity is much enhanced!)

- start w. atom in $|g_-\rangle$
- laser off: $\Omega_+ = \Omega_0$ (cpl. to vacuum):
  - $\Rightarrow$ much smaller than Ω_- (enhanced by cavity)
- Ramp up laser on Ω_+ transition
  - $\Rightarrow$ Ω_+/Ω_- goes from ≈ 0 to $\approx \infty$ (strong laser)
- photon transf. into cavity (from there it decays into ext. mode)

How can we *prepare* atom in $|g_-\rangle$ in the first place?

Optical pumping:



- Atoms initially either in $|g_-\rangle$ or $|g_+\rangle$ (statistical mixture)
- Shine light on Ω_+ transition (σ_+ - polarized):
  atom gets excited into $|e_0\rangle$ and decays randomly either into $|g_-\rangle$ or $|g_+\rangle$.
- After some time, all atoms are in $|g_+\rangle$
- Absorption of atomic gas $\Leftrightarrow$ fraction of atoms in $|g_-\rangle$
- $|g_+\rangle$ "dark state" — atom stays there