Interaction of quantized light w/ two-level system: <u>Rabi oscillations</u>

<u>No detuning ("resonant"):</u>

Cavity in Fock state: $|\psi(t)\rangle = \cos\left(\frac{\Omega_n t}{2}\right)|e,n\rangle + \sin\left(\frac{\Omega_n t}{2}\right)|g,n+1\rangle$

with n-photon Rabi frequency $\Omega_n = \sqrt{n+1}\,\Omega_0$ $\left(\Omega_0 = \frac{2d\mathcal{E}_0}{\hbar}\right)$

Cavity in arbitrary state $\sum c_n|n\rangle$, atom in $|e\rangle$:

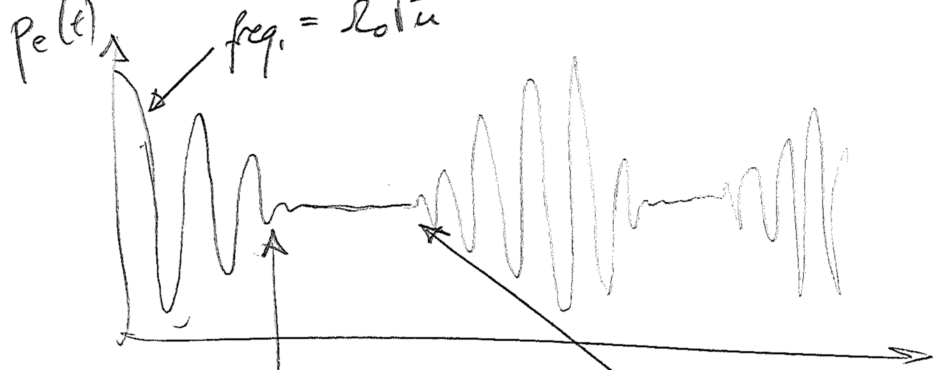
$$|\psi(t)\rangle = \sum_n c_n \left[\cos\frac{\Omega_0\sqrt{n+1}}{2}t\,|e,n\rangle + \sin\frac{\Omega_0\sqrt{n+1}}{2}t\,|g,n+1\rangle\right]$$

Probability $p_e(t)$ for atom in $|e\rangle$:

$$p_e(t) = \sum_{n=0}^{\infty}|c_n|^2\cos^2\left(\frac{\Omega_0\sqrt{n+1}}{2}t\right) = \sum_{n=0}^{\infty}|c_n|^2\,\frac{1+\cos(\Omega_0\sqrt{n+1}\,t)}{2}$$

$\Rightarrow$ $p_e(t)$ is sum of many incommensurate Rabi oscillations!

(Numerical) observation for coherent light $|\alpha\rangle = \sum c_n|n\rangle$:



<u>collapse</u> – could be explained classically (damping)

<u>revival</u> of Rabi oscillations – cannot be explained classically!

Explanation: For coherent light:

$$\bar{n} = |\alpha|^2 \; ; \quad \Delta n = \sqrt{\bar{n}} \quad (\text{Poisson distribution})$$

→ n around $\bar{n}$ with width Δn.

$$\cos(\Omega_0 \sqrt{n+1}\, t) = \cos\left(\Omega_0 \sqrt{\bar{n}+1} \sqrt{1+\frac{\delta n}{\bar{n}+1}}\right) \approx \cos\left(\underbrace{\Omega_0\sqrt{\bar{n}+1}}_{=:\Omega}\left(1+\frac{\delta n}{2\bar{n}+2}\right)\right)$$

$$= \cos \Omega \left(1+\frac{\delta n}{2\bar{n}+2}\right) t \qquad (\delta n = n - \bar{n})$$

→ Initial oscillation w/ freq. $\Omega = \Omega_0 \sqrt{\bar{n}+1}$

→ Oscillators get out of phase at rate $\frac{\delta n}{2\bar{n}+2} t$

Collapse: After time $\Omega \cdot \frac{\Delta n}{2\bar{n}} t \sim \pi$: oscillators in range $n \pm \Delta n$ completely out of phase → $\Omega_0 \sqrt{\bar{n}} \frac{\sqrt{\bar{n}}}{2\bar{n}} t \sim \pi$

⟹ Collapse after $t_c \sim \frac{2\pi}{\Omega_0}$ (independent of $\bar{n}$!)

→ approx. $\sqrt{\bar{n}}$ oscillations before collapse.

Revival: Oscillators back in phase if $\Omega \cdot \frac{1}{2\bar{n}} t \sim 2\pi$, i.e.

⟹ Revival after $t = \frac{4\pi}{\Omega_0} \sqrt{\bar{n}}$

Classical limit: $\bar{n}$ very large, Ω_0 very small

⟹ $\Omega = \Omega_0 \sqrt{\bar{n}}$; $\sqrt{\bar{n}} \gg 1$ oscillations

Experimental realization: Rydberg atoms in micro-wave cavities (Haroche group; Nobel prize 2012)

* Two-level system:

Rubidium atom in circular Rydberg state $|nC\rangle$:

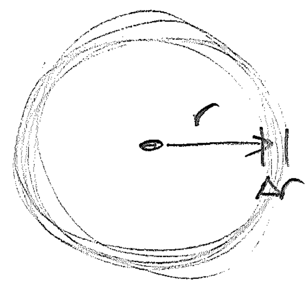
- 1 valence electron – Hydrogen-like
- valence electron highly excited:

principal q. number $n \simeq 50$

max. angular momentum $l = n-1$ ($\Rightarrow$ circular),

magnetic q. number $m = l$.

Semi-classical orbit of electron:



Bohr radius, ~0,5 Å

circular orbit w/ radius $r = a_0 n^2$,

width of orbit $\Delta r/r \sim 1/\sqrt{2n}$

States: $|g\rangle = |50C\rangle$

$|e\rangle = |51C\rangle$

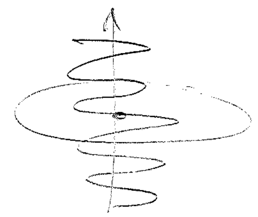
Energies: $E_n = -\frac{R}{n^2} \Longrightarrow \hbar\omega_{eg} = \frac{2R}{n^3}$

$\Longrightarrow \omega_{eg}/2\pi = 51\,\text{GHz}$ ($\lambda = 5.9\,\text{mm}$)

Matrix element of dipole transition $\langle nC|\vec{r}\,|n+1,C\rangle$:

Spatial overlap of $|nC\rangle$ & $|n+1,C\rangle$ almost 1 ($\frac{r_{n+1}-r_n}{r_n} \sim \frac{2}{n}$; $\frac{\Delta r_n}{r_n} \sim \frac{1}{\sqrt{2n}}$),
and $\Delta l = 1$ (i.e., right symmetry for non-zero matr. element)

$$d = a_0 e n^2/\sqrt{2} = 1776\, e a_0$$

(for orbit in xy-Plane & light along z: )

Selection rule for dipole transitions (from symmetries):

$$\Delta l = \pm 1;\quad \Delta m = \underset{\text{field in } z}{0},\ \underset{\text{field in } xy}{\pm 1}$$

Problem: Many degenerate levels for given n ⇒ small perturbations induce transitions $|n, l=n-1\rangle \longrightarrow |n, l=n-2\rangle$ etc.!

Solution: Apply electric field along z:

⇒ energy splitting of degen. states (linear Stark effect)

⇒ $|nC\rangle = |n, l=n-1, m=l\rangle$ non-degenerate w/ $|n, l=n-2, m=l-1\rangle$

⇒ protection against losses (together w/ selection rules)

Quadratic Stark effect: Ext. field E polarizes atom, $D = p\cdot E$ ⇒

⇒ energy shift $\propto pE^2$. For $|nC\rangle$, p dep. on n

⇒ Can be used to detune transition $|nC\rangle \leftrightarrow |n+1,C\rangle$ by applying z field.

Stability of Rydberg states:

Only possible decay: $|51C\rangle \to |50C\rangle$ etc. (selection rules)

Quasi-class. theory: accel. charge radiates, decay time ~ time to emit $E_{51}-E_{50}$. (Note: circular orbits are least accel. → most stable!)

Result: $\Gamma_n = \frac{2}{3}\frac{\alpha^3}{n^2}\omega_{eg}$; $\Gamma_n = \frac{1}{T_n}$: inverse lifetime

$\Gamma_{51} = 28s^{-1} \Rightarrow T_{51} = 36\,ms$

"quality factor" $Q = \frac{1/\Gamma}{1/\omega_{eg}} \simeq 10^{10}$ ($\hat{=}$ # of oscillations before decay)

Note: Thermal background radiation reduces lifetime by factor $(1+n_{th})$ [n_{th} = # thermal photons, cf. Rabi freq. $\Omega_n = \Omega_0\sqrt{1+n}$!)

$\Rightarrow$ shielding / cooling important!

Which n should we choose?

Large n:

pro

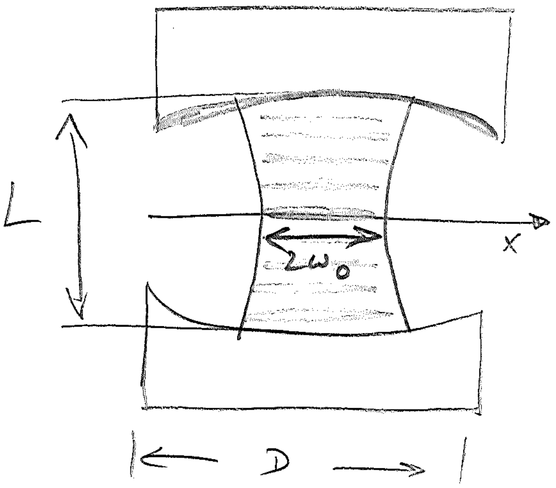
- larger lifetime
- larger dipole moment

con

- polarizability large $\Rightarrow$ more sensitive to background fields
- thermal background radiation at ω_{eg} increases.

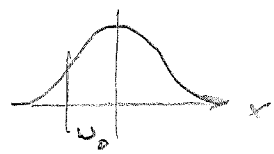
$\Rightarrow$ $\underline{n = 50, 51}$.

The cavity: "Fabry-Perot cavity"



* superconducting, Niobium mirrors @ 0.6K
* spherical mirror, radius $r = 40$ mm
* diameter $D = 50$ mm
* distance $L = 27$ mm
* width $w_0 = 7.5$ mm

Mode: TEM_{900} : transverse standing wave w/ 9 anti-nodes along z axis, angular momentum 0, and Gaussian radial intensity profile:



L can be pre-tuned $\Rightarrow$ cavity resonant w/ atom, $\omega_r = \omega_{eg}$!

Mode volume: $V = \frac{\pi}{4} w_0^2 L \approx 700\ \text{mm}^3$

$$\Rightarrow \mathcal{E}_0 = \sqrt{\frac{\hbar \omega_e}{2 \varepsilon_0 V}} = 1.5 \cdot 10^{-3}\ \text{V/m}$$

$\Rightarrow$ macroscopic value of field/photon!

Quality factor of mirrors: $Q = 3 \cdot 10^8$ (new cavities are 10^{10})
(i.e.: light is reflected $3 \cdot 10^8$ times before lost)

$\Rightarrow$ lifetime of photon in cavity $T = \frac{Q}{\omega_r} \approx 1$ ms

Avg. photon # in cavity @ 0.6K: 0.06 thermal photons

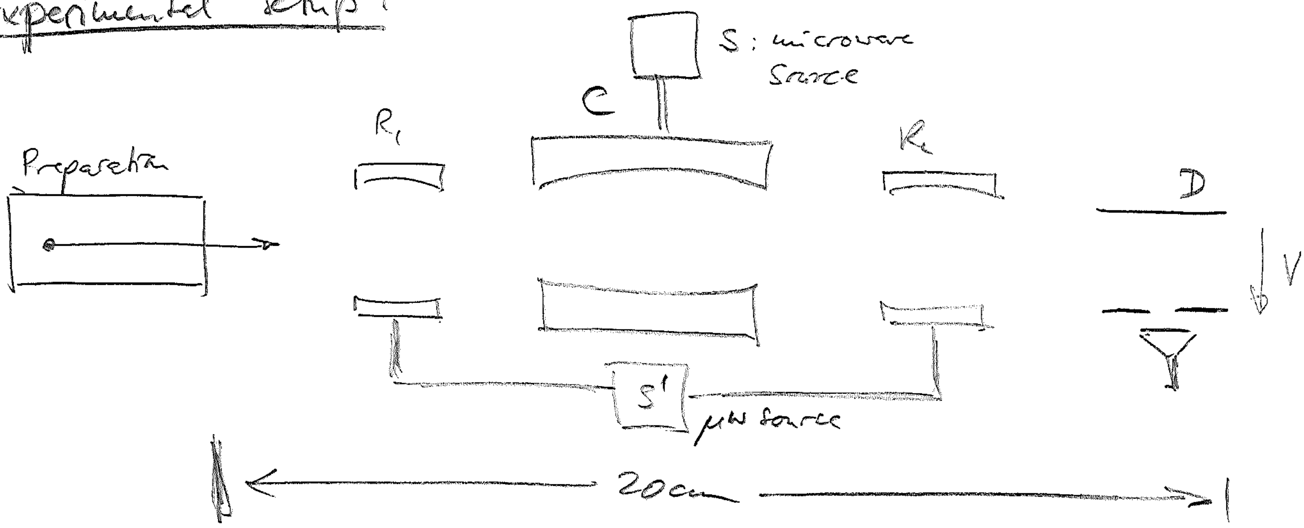
Atom – Cavity coupling

$$\left.\begin{array}{l} d = 1776\ e a_0 \\ \mathcal{E}_0 = 1{,}5\cdot 10^{-3}\ \mathrm{V/m} \end{array}\right\} \Longrightarrow \Omega_0/2\pi = 50\,\mathrm{kHz} \mathrel{\hat{=}} 20\,\mu s$$

Time scales:

field: $\omega_{eg}/2\pi$	interaction: $\Omega_0/2\pi$	cavity decay Γ_{cavity}	atom decay Γ_a	in Hz
$5\cdot 10^{10}$	$5\cdot 10^{4}$	$1\cdot 10^{3}$	$3\cdot 10^{1}$	

Experimental setup:



Preparation: Prepares pulses w/ ~0.1 atom/pulse with well-defined speed. (Details: Book by Haroche & Raimond)

- Speed 140 – 600 m/s ± 2 m/s (Note: 2π-interaction in cavity ~400 m/s)
- time to pass system ~ 1 ms
- spacing of consec. atoms flexible
- Works by selectively exciting atoms w/ specific speed in atom beam

Atoms prepared in $|e\rangle$ or $|g\rangle$.

<u>R_1/R_2</u>: "Bad cavity" – applies <u>classical field</u> to atom.

→ Rotates spin via Ramsey pulse,

$$\pi\text{-pulse} \propto e^{i\pi\sigma_x} \Rightarrow \text{Rotates} \quad |e\rangle \to |g\rangle, \quad |g\rangle \to -|e\rangle$$

$$\pi/2\text{-pulse} \propto e^{i\pi/2\,\sigma_x} \Rightarrow \text{Rotates} \quad |e\rangle \to (|e\rangle + |g\rangle)/\sqrt{2}, \quad |g\rangle \to (|e\rangle - |g\rangle)/\sqrt{2}$$

Used to prepare atom & to rotate for read-out.

<u>C</u>: Cavity. Interaction time tuned by speed or by detuning atom via electric field in z direction.

Cavity coupled to (weak) microwave field via small hole in cavity.

<u>D</u>: Detection by ionization in el. field & amplifying electron.

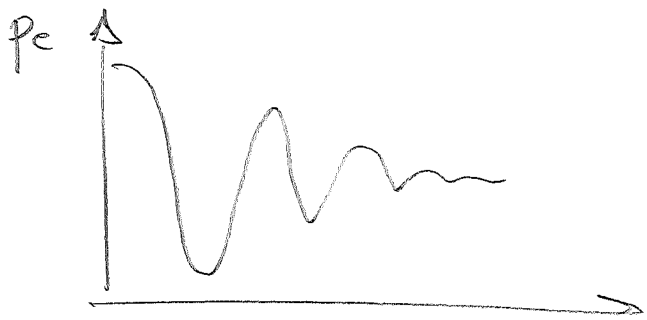
Ionizing field ⟺ Rydberg state ($n=50$: 145 V/cm; $n=51$: 134 V/cm)

Since # atoms ~ 0.1, postselection of results is necessary.

Experiment 1: Rabi oscillations

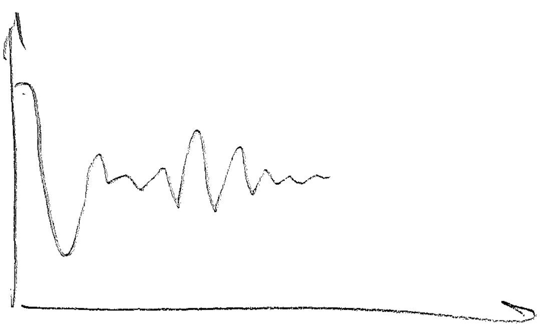
- Atom prepared in $|e\rangle$.
- Cavity empty : $|0\rangle$
- No detuning : $\omega_c = \omega_{eg}$
- No pulse in R_1
- No pulse in R_2: atom measured in $\{|e\rangle, |g\rangle\}$ basis.

$\Rightarrow$ Vacuum Rabi oscillations:



(decay due to exp. imperfection)

- Cavity initialized in weak coherent field (inj. from microwave source):



Fourier-Trafo