Lecture "Quantum Optics" — Exercise Sheet #12

Problem 1 (easy)

Express the Lamb-Dicke parameter $\eta_0 = k_\ell a_0$ in terms of the ratio of the recoil energy acquired by an ion at rest by absorbing a photon of frequency ω_ℓ and the energy $\hbar\nu$ of a phonon. What regime does the Lamb-Dicke regime correspond to, and what does this mean for the chance of an absorption process to excite a sideband transition?
Compute the Lamb-Dicke parameter for a Ca^+ and Be^+ ion, for an optical transition ($\lambda_\ell = 500\,\mathrm{nm}$), and $\nu/2\pi = 1\,\mathrm{MHz}$.

Problem 2 (easy)

In the lecture, we have discussed how shining a travelling-wave laser beam on a trapped ion induces a coupling between the internal state of a trapped ion and its vibration. Now consider the same scenario, but with a standing wave laser,

$$E(z) = \mathcal{E}\sin(k_\ell z + \phi)e^{-i\omega_\ell t} + \text{h.c.}\ .$$

Show that in this case, the interaction is of the form

$$H_{\mathrm{cpl}} = -i\hbar\frac{\Omega_\ell}{2}e^{-i\omega_\ell t}\big[\sin\phi + \eta_0\cos\phi\,(a^\dagger + a)\big]\sigma^+ + \text{h.c.}\ ,$$

where Ω_ℓ is determined from $\mathcal{E}$ and the dipole moment of the atomic transition.

Problem 3 (medium)

Generalize the derivation of how applying a laser induces a coupling between the internal levels and the vibration of a trapped ion to the case where two lasers inducing a Raman transition via a third level are being used instead of a direct transition.
The idea is to follow closely the derivation of Raman transitions given in Lecture 11 (pg. 88 and 89). To this end, consider two lasers which are strongly detuned from the third state, so that (like in the lecture) we can assume that the third level is almost not populated to solve the coupled differential equations. The easiest way to carry out the derivation is then to observe that the position-dependence of the light field can be effectively described by replacing $\Omega_\pm \to \Omega_\pm e^{\eta_\pm(a^\dagger+a)}$, which allows to follow exactly the same steps as in Lecture 11. (Note that the derivation in the lecture was for *real* $\Omega_\pm$, so you need to add complex conjugates in some places.)
Show that the effective interaction between the two relevant levels of the ion and the vibration is the same as the one derived in Lecture 12, where now the effective Ω and η depend on properties of both laser beams. Discuss how the result depends on (i) the frequencies of the two lasers (and thus the energy difference of the two levels) and (ii) the angles of the two Raman laser beams relative to the trap axis. Show that even if the two levels are degenerate, changing the angles allows to tune the effective Lamb-Dicke parameter. For which settings does one get the largest/smallest value?