Lecture "Quantum Optics" — Exercise Sheet #7

Problem 1 (easy)

- Derive the radius and energy of the n-th Rydberg state from semi-classical considerations, using the Coulomb law together with the fact that the orbital angular momentum is quantized in multiples of $\hbar$.
- Use Heisenberg's uncertainty to derive the width of the orbit, $\Delta r/r = 1/\sqrt{2n}$.
- Derive a classical approximation for the energy shift of a Rydberg atom in an electric field (the quadratic Stark effect), by considering the atom as a charged center + a charged ring of radius r, subject to an electric field. How do the obtained shifts (for fields far enough from the ionization threshold) compare to the transition frequencies?

Problem 2 (tricky)

- Let S be a hermitian operator, and let $|\psi_1\rangle$ and $|\psi_2\rangle$ be eigenstates of S with eigenvalues s_1 and s_2, respectively. Further, let Q such that $[Q, S] = qQ$ (with q some number). Show that $\langle\psi_2|Q|\psi_1\rangle$ can only be non-zero if $s_1 - s_2 = q$.
- Use this to derive the selection rules for the magnetic quantum number, $S = L_z$, for electric dipole transitions, for *(i)* $Q = z$ and *(ii)* $Q = x \pm iy$ (with $\boldsymbol{r} = (x, y, z)$ the position operator).
- Derive the corresponding selection rule for $\boldsymbol{L}^2$. To this end, first show that $[\boldsymbol{L}^2, [\boldsymbol{L}^2, \boldsymbol{r}]] = q\{\boldsymbol{r}, \boldsymbol{L}^2\}$, which can then be used to derive the selection rule.

Problem 3 (medium)

The aim of this problem is to construct protocols for the atom+cavity scheme of Haroche described in the lecture.

- Describe a scheme (consisting of an initial Rabi pulse in the cavity R1, an interaction time in the cavity C, and a final pulse in R2) which, starting from the atom in $|e\rangle$ and the cavity in the vacuum, creates a maximally entanged state $(|e, 0\rangle + |g, 1\rangle/\sqrt{2}$ of the atom and the cavity.
- Assume that the atom is initially in $|e\rangle$ and the cavity is initially in an unknown state $\alpha|0\rangle + \beta|1\rangle$. Describe a scheme which allows to transfer the state of the cavity onto the atom, leaving the atom in the state $\alpha|g\rangle - \beta|e\rangle$. What do we have to change if we instead want the atom to be in the state $\alpha|g\rangle + \beta|e\rangle$, or $\beta|g\rangle + \alpha|e\rangle$?
- Combine the two preceding schemes to create a pair of entangled atoms in the state $(|e, g\rangle - |g, e\rangle)/\sqrt{2}$, by using the cavity to "mediate" the entanglement. At the end of the protocol, the cavity should not be entangled with the rest of the system, and be left in a well-defined state.