Lecture "Quantum Optics" — Exercise Sheet #4

Problem 1 (medium difficult, but very lengthy)

In Lecture 4, we have determined the sensitivity of the interferometer where we input a coherent beam $|\alpha\rangle_a$ in mode a and the vacuum $|0\rangle_b$ in mode b. The aim of this problem is to study how squeezed inputs can increase the sensitivity of an interferometer.

1. Consider the situation where instead of $|0\rangle_b$, we feed a squeezed vacuum $S(re^{2i\chi})|0\rangle_b$ ($r \geq 0$) into the second mode. What is the optimal sensitivity?
2. Compare this to the optimal sensitivity obtained when feeding a squeezed input $D(\alpha)S(re^{2i\chi})|0\rangle_a$ into mode a and the vacuum $|0\rangle_b$ in mode b.
3. Compare the two results. Does it matter which input we squeeze?

Note: You need to choose the phase of α and $e^{2i\chi}$ such as to minimize the noise. In the regime of interest, $|\alpha|^2 \gg e^r$. (Typical squeezing parameters which can be realized are around $e^r \approx 10$, while the number of photons $|\alpha|^2$ even at moderate laser powers is far above that – e.g., 1mW corresponds to 4×10^{15} photons per second at 800nm.)

In order to determine the optimal sensitivity, you can either plot the sensitivity as a function of the phase difference ϕ, or you can neglect all but the leading order in $|\alpha|$ and derive an analytical expression.

Squeezed light is used for instance to enhance the resolution of gravitational wave detectors. You can find more details e.g. in `http://arxiv.org/abs/1310.0383` – it is instructive to relate the orders of magnitude of the involved quantities (laser power, squeezing, time to be resolved, and desired sensitivity).

Problem 2 (short, but a bit tricky)

Show that the following setup provides a way to apply the displacement operator $D_a(\alpha)$ to an arbitrary input state $|\psi\rangle_a$: Mix the state $|\psi\rangle_a$ with a very strong coherent beam $|\beta\rangle_b$ on a very weakly mixing beam splitter $U(\epsilon) = \exp(\epsilon(a^\dagger b - b^\dagger a))$ ($\epsilon \ll 1$), with $\epsilon\beta = \alpha$. (The result is obtained after some transformations by taking the limit $\epsilon \to 0$, while keeping $\epsilon\beta = \alpha$ constant.)

Problem 3 (medium)

Consider two squeezed states $|a\rangle_a = S(r)|0\rangle_a$ and $|b\rangle_b = S(-r)|0\rangle_b$, for some $r > 0$. What are the variances ΔQ_a, ΔQ_b ΔP_a, and ΔP_b (with $Q_a = (a^\dagger + a)/\sqrt{2}$, $P_a = i(a^\dagger + a)/\sqrt{2}$, and correspondingly for mode b)? Draw the corresponding phase space distribution for modes a and b.

We now mix the two modes on a balanced beamsplitter and obtain $a' = (a + b)/\sqrt{2}$, $b' = (a - b)/\sqrt{2}$. The output state cannot be described for mode a' and b' independently any more; it corresponds to a distribution in a four-dimensional phase space with coordinates $Q_{a'}$, $Q_{b'}$, $P_{a'}$, and $P_{b'}$.

- What are the variances $\Delta Q_{a'}$, $\Delta Q_{b'}$, $\Delta P_{a'}$, and $\Delta P_{b'}$?
- What are the variances of $\frac{1}{\sqrt{2}}(Q_{a'} + Q_{b'})$ and $\frac{1}{\sqrt{2}}(P_{a'} - P_{b'})$? How can this be interpreted? Why does this not violate the uncertainty relation?
- How does the phase space distribution in the two-dimensional phase space spanned by $Q_{a'}$ and $Q_{b'}$) look like?