non-relativistic QED problem:

* nuclei + electrons, Schrödinger equation
* Born-Oppenheimer approx.:

  Solve electron problem in field of nuclei (→ ground state!)

  nuclei see effective electron density

→ configuration of molecules, crystals, CM systems.

→ mechanical problems (stiffness etc.): similar

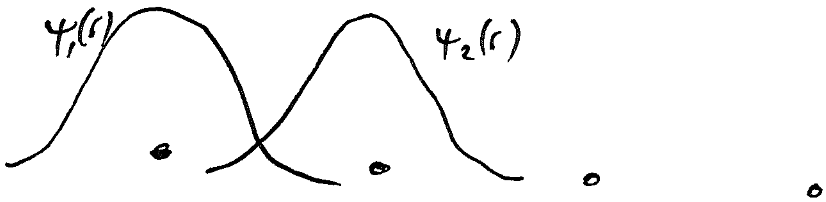
* conductivity, magnetism, etc:

  property of electrons.

* most electrons: tightly bound to "their" nucleus.

  only some valence electrons might move.

* tight-binding model: even those strongly bound to "their" nucleus → localized eigenstates.



2nd quantization: $|\Omega\rangle$ = "vacuum"

a_i^+: create electron in state $\psi_i(\underline{r})$ ~~spin σ~~

$$a_i^+ a_{i'}^+ |\Omega\rangle \equiv \psi_i(\underline{r}_1)\,\psi_{i'}(\underline{r}_2) - \dots$$

ψ_1 and ψ_2 overlap $\longrightarrow$ electron can tunnel betw. nodes!

$$H = -t \sum_i a_i^\dagger a_{i+1} + a_{i+1}^\dagger a_i \qquad (1D)$$

general lattice:

$$H = -t \sum_{\langle i,j \rangle} a_i^\dagger a_j + a_j^\dagger a_i$$

Notation for "adjacent sites in the lattice"

$\Rightarrow$ $\boxed{\text{free fermions} \Rightarrow \text{solved by Bloch waves!}}$

* now let's add spin: $a_{i\uparrow}, a_{i\downarrow}$ $\quad (a_{i,\sigma})$.

Moreover, if two fermions are at the same site, they experience Coulomb repulsion:

$$H = -t \sum_{\langle i,j \rangle, \sigma} \left(a_{i\sigma}^\dagger a_{j\sigma} + a_{j\sigma}^\dagger a_{i\sigma} \right) + U \sum_i n_{i,\uparrow} n_{i,\downarrow}$$

$$n_{i,\uparrow} = a_{i\uparrow}^\dagger a_{i\uparrow}$$

$\boxed{\text{"Hubbard model"}}$

* Special limit of the Hubbard model:

half-filling = 1 electron per site

$U \gg t$ (tight binding)

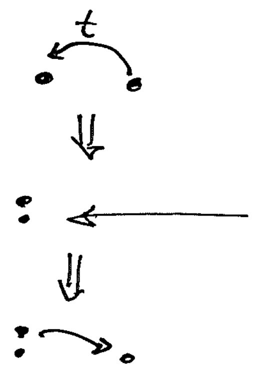
~~Ground state~~ .

$$H = \underbrace{U \sum_i n_{i\uparrow} n_{i\downarrow}}_{H_0} - \underbrace{t \sum \left(a_{i\sigma}^{+} a_{j\sigma} + a_{j\sigma}^{+} a_{i\sigma} \right)}_{t \cdot V}$$

ground state of H_0: 1 electron per site.

2nd order perturbation theory:

V: virtual process:



only possible if singlet!

Energy correction from virtual process:

$$-t^2 \langle \cdot,\cdot | V | :, \rangle \frac{1}{\langle :,\_ | H | :,\_ \rangle - \langle \cdot\cdot | H | \cdot\cdot \rangle} \langle :,\_ | V | \cdot\cdot \rangle$$

$$= -\frac{t^2}{U}$$

i.e.: singlet energy lower that triplet energy (by $\frac{t^2}{u}$).

$$H_{eff} = -\sum_{\langle i,j\rangle}' |\psi^-\rangle\langle\psi^-|_{i,j}$$

$$|\psi^-\rangle_{ij} = |\uparrow\rangle_i|\downarrow\rangle_j - |\downarrow\rangle_i|\uparrow\rangle_j$$

in <u>effective space</u> of singly occupied sites!

<u>Note:</u> $|\psi^-\rangle\langle\psi^-|_{ij} = -\left(\vec{S}_i\cdot\vec{S}_j - \frac{1}{4}\right)$

$\Rightarrow$ effective <u>Heisenberg model</u>:

$$\boxed{H = \sum_{\langle i,j\rangle}' \vec{S}_i\cdot\vec{S}_j}$$

~~"Problem" with these models:~~

Ground state of such models are <u>complicated</u>:

$\vec{S}_i\cdot\vec{S}_j$ wants <u>all pairs</u> to be in a singlet state

$\longrightarrow$ impossible! (<u>monogamy</u> of entanglement)

Similar problem for <u>Hubbard model</u>:

Strong correlations $\Longleftrightarrow$ perturbative methods around a "simple" state (like a product state, Bloch waves) will not work!

<u>General scenario</u> which we study:

Notation: $|\uparrow\rangle \equiv |0\rangle, \quad |\downarrow\rangle = |1\rangle$

$$|i\rangle \otimes |j\rangle \equiv |i\rangle|j\rangle \equiv |i,j\rangle \equiv |ij\rangle$$

Focus for now on <u>spin systems</u> ($\mathbb{C}^d$ at each site).

We consider <u>lattice systems</u> with <u>local interactions</u>

length N $\longrightarrow$ $\bullet \quad \overset{\mathbb{C}^d}{\circ} \underset{h_{i,i+1}}{\text{———}} \overset{\mathbb{C}^d}{\circ} \quad \circ \quad \bullet$

$(\mathbb{C}^d)^{\otimes N}$

spatially local: 2-body, 3-body, …, or quickly decaying

$$h_{i;i+1} \equiv \mathbb{1} \otimes h_{i;i+1} \otimes \mathbb{1}$$

$$H = \sum_i h_{i;i+1}$$

Focus on <u>ground state properties</u>. $|\psi\rangle$

This will be the ~~b~~ core of the lecture.

We want to know:

i) Energy of GS: $\langle\psi|H|\psi\rangle$

ii) Local observables $\langle\psi|O_i|\psi\rangle \equiv \langle\psi|\mathbb{1}\otimes O_i \otimes \mathbb{1}|\psi\rangle$

iii) correlation functions $\langle\psi|A_i \otimes B_j|\psi\rangle$

iv) and more?

~~Also~~ <u>Beyond GS</u>:

* dynamics, thermal states, …
* what are conseq. of strong corr. ($\to$ topo. order)
* how are different properties (spec., correl.) related?
* how do perturbations propagate?

How can we describe G.S. of QMB systems w/ strong interactions? (6)

$$|\psi\rangle = \sum_{i_1,\dots,i_N=1}^{d} c_{i_1,\dots,i_N} \, |i_1,\dots,i_N\rangle$$

d^N complex parameters $\left((\mathbb{C}^d)^{\otimes N} \cong \mathbb{C}^{(d^N)} \right)$

$\to$ infeasible for $N \gtrsim 30$ unless c has "nice" structure.

E.g., product state

$$|\psi\rangle = \left(a_0^1 |0\rangle + a_1^1 |1\rangle \right) \otimes \dots \otimes \left(a_0^N |0\rangle + a_1^N |1\rangle \right)$$

$$\Longleftrightarrow \boxed{c_{i_1,\dots,i_N} = a^1_{i_1} \cdots a^N_{i_N}}$$

$2N$ parameters!

But is there hope for strongly corr. ground states?..

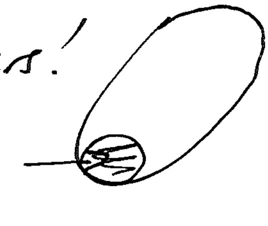
G.S. is specified by Hamiltonian

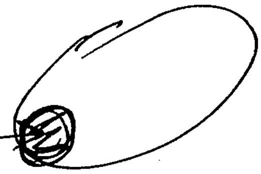
$$H = \underbrace{\sum_{\langle i,j \rangle}}_{N^2 \text{ terms}} h_{ij}$$

h_{ij}: $L(\mathbb{C}^d \otimes \mathbb{C}^d)$: d^4 parameters

$\Rightarrow$ H specified by $\sim N^2 d^4$ parameters!

$\to$ feasible for very large N!



"physical corner" of Hilbert space. 

How can we characterize the physical corner of Hilbert space?

→ Entanglement properties!