Lecture "Analytical and Numerical Methods for Quantum Many-Body Systems from a Quantum Information Perspective" — Exercise Sheet #6

1. If you haven't done so in Problem 3 on the previous exercise sheet (#5), convince yourself that the two representations of the cluster state – either as the joint eigenstate of $S_i = X_i \otimes (\bigotimes Z_j)$, or as the state obtained by starting from $|+\rangle^{\otimes N}$ and applying CPHASE=CZ gates along each edge – describe the same state.

2. Verify that Pauli errors can be moved through CZ gates. This is, if the initial state is $(P \otimes Q)|\psi_{\text{in}}\rangle$, where P and Q are Paulis, then $CZ(P \otimes Q)|\psi_{\text{in}}\rangle = (R \otimes S)CZ|\psi_{\text{in}}\rangle$, with R and S again Paulis.

3. Verify that the measurement pattern described in the lecture to implement a CZ gate (where there are two extra sites between the two "target" sites) in measurement based computation works as claimed.

4. Clifford circuits are generated by the gates $S = \begin{pmatrix} 1 & \\ & i \end{pmatrix}$, $H = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}/\sqrt{2}$, and $CZ = \text{diag}(1, 1, 1, -1)$. Show that in measurement based computation, any Clifford circuit can be implemented without using adaptive measurements, i.e., it can be implemented in a single parallel measurement layer.

5. In the last exercise sheet (Problem 3, Sheet 5) we have shown how to write the cluster state as a PEPS. The aim of this problem is to understand measurement based computation using this PEPS representation. (This is based on http://arxiv.org/abs/quant-ph/0311130.) In case you attended the quantum information course, it will be very helpful to think of this in terms of teleportation (you can also read up on this on the Wikipedia site on "Quantum Teleportation")

   - First, understand the action of a Z measurement. (Note that a projective Z measurement on the physical level leads to projections in the Z basis applied to the individual *virtual* sites. On the other hand, projecting one end of an entangled state onto a pure state can be described as removing the entangled state and putting a certain pure state on the other end.)
   - Second, understand the way one-qubit operations work by considering the PEPS representation of a one-dimensional cluster state. (Note that in a 1D cluster, projecting on the physical level in the XY plane translates into projecting onto some maximally entangled state on the virtual level, and that this is similar to what happens in teleportation.) Also understand the way in which the read-out works.
   - Finally, consider the way two-qubit operations work using PEPS.

6. Consider a one-dimensional classical Hamiltonian

$$H(s_1, \ldots, s_N) = h_{1,2}(s_1, s_2) + h_{2,3}(s_2, s_3) + \cdots + h_{N-1,N}(s_{N-1}, s_N) \ ,$$

   where $s_k = 1, \ldots, d$, and $h_{k,k+1}(s_k, s_{k+1}) \in \mathbb{R}$ is the classical two-body Hamiltonian. Show that the ground state of H can be found as follows: Denote by $E_k(s_k)$ the minimal energy of all Hamiltonian terms left of k (i.e., $h_{1,2}, \ldots, h_{k-1,k}$) as a function of s_k (optimized over $s_1, \ldots, s_{k-1}$). Show that $E_1(s_1)$ can be computed efficiently, and that given $E_k(s_k)$, one can compute $E_{k+1}(s_{k+1})$. Show that together, this allows to find the ground state of H. (This method is known as "dynamic programming".)